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PROCESSING IRRADIANCE MEASUREMENT DATA IN THE FIELD-OF-VIEW DOMAIN

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ABSTRACT

The sensing of spatial imagery and spectral signatures involves the measurement of radiance. However detectors measure irradiance and the radiance must be inferred from that. This paper reports a method of processing irradiance measurement data to obtain radiance in a manner so as to reduce the errors caused by the unknown nonuniform illumination of the instantaneous-field-of-view (IFOV) of the individual optical detectors in an array of such detectors comprising an imaging or non-imaging sensor. Significant error and image distortion reductions seem to be achievable with this new approach.

The derivative relationship between radiance and irradiance is implemented through its mathematical definition as a limit in a new coordinate space called the field-of-view domain. An appropriately chosen set of nested clusters of detectors surrounding and including the one of interest provides the data for forming the limit that is taken to an IFOV of zero angular width.

Key words: Nonuniform illumination, instantaneous-field-of-view, field-of-view domain, derivative-as-limit, detector array

INTRODUCTION

The measurement of radiance underlies many operations in remote sensing, such as determining spectral signatures (e.g. via measuring spectral directional reflectance factor) and sensing spatial imagery. Since detectors measure irradiance, the radiance is inferred and not directly measured. This paper is concerned with a method for dealing with a problem in the processing of irradiance measurement data in order to determine radiance that does not appear to be directly addressed in the literature. That is, the errors produced by the unknown nonuniform illumination of the instantaneous-field-of-view (IFOV) of the individual optical detectors in an array of such detectors comprising a sensor.

These considerations are applicable to both passive and active sensors and to sensing via reflected and/or emitted radiance. They are applicable to almost all detectors in the visible, infrared and ultraviolet bands that are designed to respond to the total incident optical power in whatever wavelength bandwidths they are sensitive.

Irradiance (E) and radiance (L) are fundamentally related through the mathematical derivative relation

(1) $\quad L = dE \,/\, d\Sigma$

(where Σ = projected solid-angle) and its inverse (integral) relation

(2) $\quad E = \int L \, d\Sigma$

(Nicodemus, Richmond, et al., 1977). This effectively leads to characterizing the illuminated detectors as averaging the unknown radiance received over the various lines-of-sight within their IFOV. When the illumination of the detector's IFOV is uniform or nearly so, there is little or no error from using a radiance/irradiance geometric relation specific to the detector geometry to determine the radiance from the detector response.

In terms of the average radiance incident on the i th detector, Lav_i, (averaged over the projected solid-angle Σ_i of the IFOV of the i th detector)

(3) $\quad Lav_i = \int L_i(\varnothing,\Omega)\, d\Sigma_i \,/ \int d\Sigma_i$

(where $\varnothing$ = polar angle and Ω = azimuth angle and $d\Sigma_i = \cos\varnothing_i \sin\varnothing_i \, d\varnothing_i \, d\Omega_i$) the irradiance illuminating the i th detector can be expressed as

(4) $\quad E_i = \Sigma_i \,(Lav_i)$.

Clearly when $L_i(\varnothing,\Omega)$ = constant then L_i can be inferred from

(5) $L_i = E_i / \Sigma_i$

However, when the unknown illumination of the IFOV is strongly nonuniform, the detector response is related to the average radiance value, and the measurement data user must consider what relationship that value has to the desired quantity. Such a desired quantity might be the peak radiance within the IFOV or the radiance along the line-of-sight through the geometric center of the IFOV or some other quantity. The choice of quantity will generally depend on what the application of the measured data will be. The difference between the desired and inferred quantities will simply be called the error. Such errors lead to image distortion. The next step is to relate the desired quantity to the average radiance measured by that detector. This paper will discuss a method for determining approximate peak and valley radiance values in symmetric cases from measurement data as well as approximate values at the detector geometric surface centers for monotonically increasing and decreasing segments of the radiance fields.

The method is based on implementing the derivative relationship between L and E through its mathematical definition as a limit in a new coordinate space called the field-of-view domain. An appropriately chosen set of nested clusters of detectors surrounding and including the central one of interest provides the data for forming the limit that is taken to an IFOV of zero angular width. If the IFOVs were essentially infinitesimal in size, and the nested clusters of these arrays were essentially a continuum of such detectors, we would expect the limiting process to produce almost exact results. Obviously, in realistic situations these conditions are not met and we can only expect approximate results, although it will be seen that such approximations may be quite good.

The usefulness of this technique is explored and significant improvement demonstrated through the use of both synthetic measurement data and airborne imagery. The synthetic data are generated by modeling the measurement process and applying that to assumed incident (on the detector) nonuniform spatial profiles of radiance beams. Two types of situations are examined that are distinguished through the use of a beamwidth (BW) parameter for the radiance defined as the angular span between two points (nearest to the beam peak but on either side of it) where the radiance is one-half the beam peak value. The two situations are defined by r = (BW)/(IFOV) greater than and less than one.

An example for which r = 2.5 shows modest absolute improvement but significant relative improvement using the new method. In this case the measurement of peak radiance of the beam is in error by about 13 percent (assuming all other error sources to be zero) of the true (initially assumed) value while the corrected (with the new method) measurement is in error by less than one percent. In a second example, where r = 0.7, the measurement of the beam's peak radiance is in error by 36.8 percent while the corrected result is only in error by 13.6 percent. So far we have found that the smaller r is: (1) the larger the measurement error and (2) the greater the improvement after correction so long as the irradiance measured in the surrounding detectors that contribute to the calculation of the limit are measured with sufficient accuracy.

The errors in the radiance measurements due to nonuniform illumination cannot be calibrated out, to our knowledge, by any current in-lab measurements performed before or after field use, because the nonuniformities in illumination: (1) for any one detector are not known in advance since they vary with the scene being viewed, (2) vary from detector to detector in the array, and (3) vary from instant to instant in all detectors if the scene varies with time for fixed observers, and if the array and the scene being observed are in relative motion.

Examination of the airborne imagery corrected with this new approach supports the idea that the method can be useful for spatial deblurring and contrast enhancement in imagery when there are rapid variations in measured radiance due to the presence of a variety of conditions e.g. fine surface features, relatively small objects and rapid changes in ground reflectance and/or emittance due to abrupt changes in ground composition. No attempt was made to include, in the correction procedure used on the imagery, methods to reduce errors from error sources other than the unknown nonuniform illumination of the IFOV.

Additional corrections made for other classes of error sources would presumably further improve the image quality. Figures 1 and 2 show an airborne image and a corrected (for an unknown nonuniform illumination) version, respectively, of a scene viewed from an altitude of ten thousand feet. An enhancement in contrast and spatial deblurring caused by the use of the derivative-as-limit technique is reasonably evident. The improvement in accuracy also is related to an improvement in spatial resolution in the sense of the McKenna equivalent spatial resolution enhancement factor, Mr, discussed in the fifth section of this paper.

II. ERRORS AND IMAGERY DISTORTIONS

There are many sources of errors and imagery distortions for remotely sensed imagery, both endogenous and exogenous, some briefly outlined in this section. External sources of

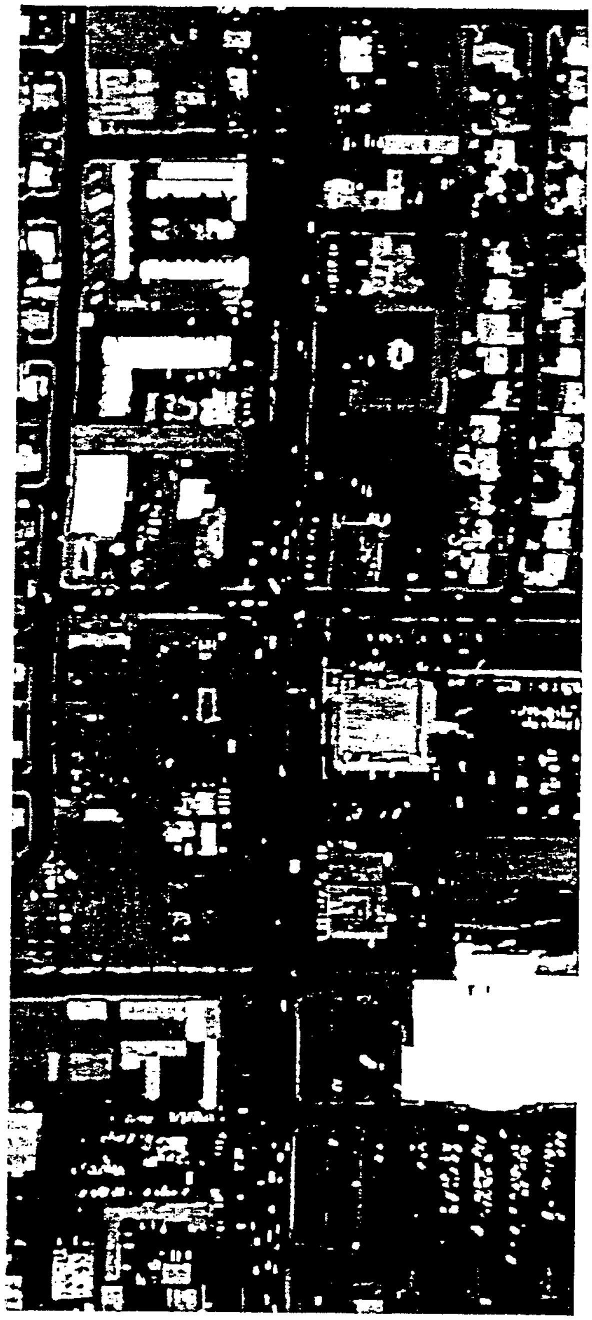

Figure 1. Uncorrected image of an urban area taken from an altitude of ten thousand feet.

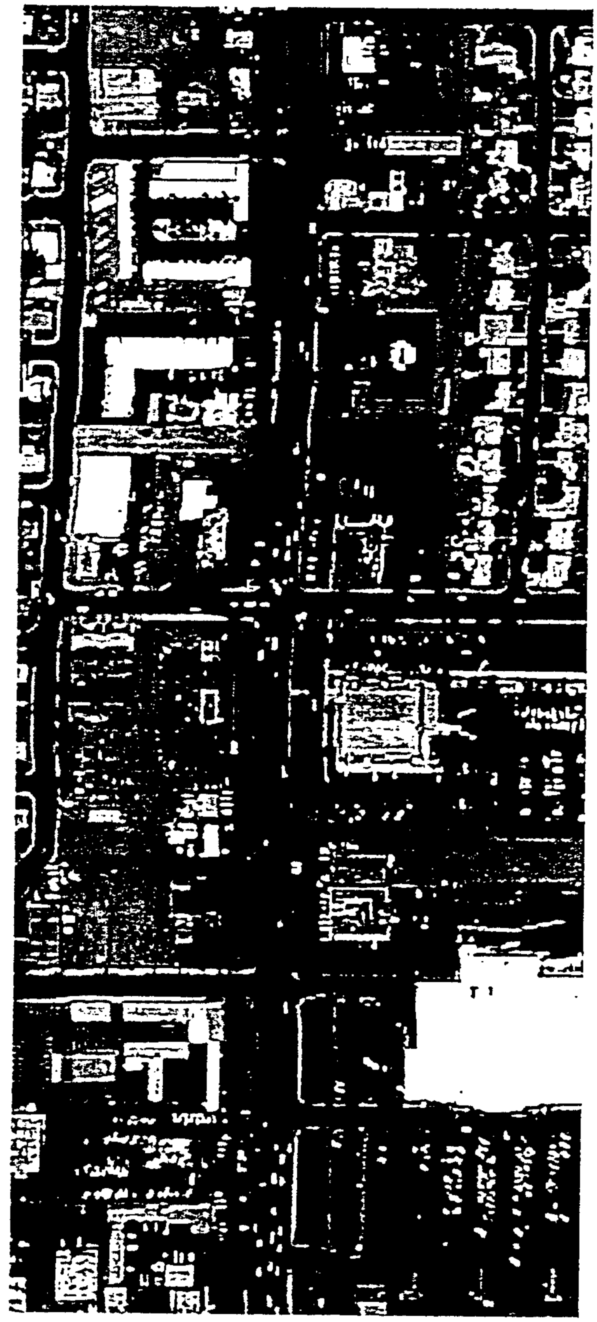

Figure 2. An altered version of Figure 1 corrected only for an unknown nonuniform illumination with a derivative-as-limit algorithm.

image errors and distortions are usually dominated because of the presence of the atmosphere. These are very much dependent on the state of the atmosphere through which the optical energy propagates as well as the wavelength bands being used. A great deal of research has gone into the development of atmospheric correction techniques (Kaufman 1989), and descriptions of new ones, as well as modifications of old ones, continue to be published at a significant rate, especially for the single-platform problem. Platform structures and equipment supporting structures on them can induce large errors under certain conditions as well (Stover 1995).

The optical system between the array of detectors and the atmosphere also contributes

significantly to the degradation of the image to be sensed by that array (Nussbaum and Phillips 1976). The elimination or attenuation of ranges of spatial frequencies are conveniently characterized by the optical transfer function of this optical system though care must be taken with this representation when it is extended to include the detectors as well with their nonlinear characteristics.

There are a number of different classes of detectors all of which have their detection performances hindered by a variety of noise mechanisms discussed in many monographs on the subject (e.g. Dereniak and Boreman 1996). Of all these mechanisms there is one that bears some slight similarity with the unknown nonuniform illumination of the IFOV problem considered in this paper. It is the spatial photoresponse nonuniformity of the detector array that can cause a severe problem in the use of infrared focal plane arrays (Holst 1995). It is often referred to as a fixed pattern noise and is created by the unintended differences in photoresponsivity of the individual detectors in the array (Schulz and Caldwell 1995).

There also are errors because of the nature of processing. For example there are errors from the digitizing of the received analog signal resulting in some lower bound on radiometric resolution. However the authors have been unable to locate in the literature a direct discussion of a source of error or image distortion considered in this study nor the proposed procedure addressing the problem. Certainly a suitably sized aperture can create a nonuniform distortion of an image, however that is a fixed structure associated with the imaging equipment; this paper is concerned with the initially unknown nonuniformity due to the scene viewed. Therefore, it does not seem likely that it can be calibrated out by some procedure before or after operational use.

Furthermore, it is normally to be expected that each detector is being illuminated, at least in part, by some different portion of the overall scene being viewed so that each would be experiencing a different form or degree of nonuniformity in its illumination. Also, individual nonuniformity would usually be altered as the element of the scene being viewed by each detector changed. For detector arrays in relative motion with respect to the scenes being observed, as usually occurs in remote sensing, the nonuniformities in illumination of the IFOVs would continue to vary in time throughout the operational use.

Strictly speaking it is not simply the unknown nonuniformity of the illumination of the IFOV that is the problem, but its character that can cause the errors of concern. For example, consider a one-dimensional case where the illumination (i.e., the magnitude of the radiance) varies linearly with angle across the IFOV of a detector bounded by angles $Ø_i$ and $Ø_j$. Thus

(6) $L_i(Ø) = a_i Ø + b_i, Ø_i \leq Ø \leq Øj$

where a_i and b_i are constants. Then, assuming $Ø \sim 0$ so that $\cos Ø \sim 1$ as is often the case,

(7)

$$E_i = \int L_i(Ø)\, dØ = (a_i / 2)\,[Ø_i^2 - Øj^2] + (b_i)\,[Ø_i - Øj]$$

Dividing eq. (7) by $[Ø_i - Øj]$ produces the average value of $L_i(Ø)$ in the range of Ø located at the midpoint between $Ø_i$ and $Øj$,

(8) $E_i / [Ø_i - Øj] = (a_i)\,([Ø_i + Øj] / 2) + (b_i) = Lav_i$

Thus, if in the application of interest all that is required is that the measured value produces the average value of radiance at the center of the pixel, no distortion or error is produced in this case.

III. ERRORS PRODUCED BY NONUNIFORM ILLUMINATION

To illustrate the possible errors produced by an unknown nonuniform illumination of the IFOV of individual detectors, we consider a small contiguous linear array (see Figure 3) illuminated by a pattern of reflected radiance described by

(9)

$$L(Ø) = \begin{cases} L^{+}(Ø) = L_0 \exp[-k\, Ø / ø], & Ø > 0 \\ L^{-}(Ø) = L_0 \exp[+k\, Ø / ø], & Ø < 0 \end{cases}$$

Here L(Ø) represents the radiance seen along the line-of-sight which is at an angle Ø with respect to the normal to the (linear) center of the face of the central detector of a linear, odd numbered array, ø represents the half-angle of the IFOV of an individual detector in the array, Lo is the peak value of the radiance, and k is a constant that will be adjusted to illustrate the beamwidth effects in the calculations.

Consider the case of the narrow beam where the BW = 0.69 (IFOV) which corresponds to an approximate value k = 1. Table 1 presents the actual (assumed) radiance values at the specified locations (as calculated from eq. (9)), the inferred data (as calculated from the average values from eq. (3) for each detector) and the percent difference between these values.

The results in Table 1 support the idea that when the beamwidth of the illuminating radiance field

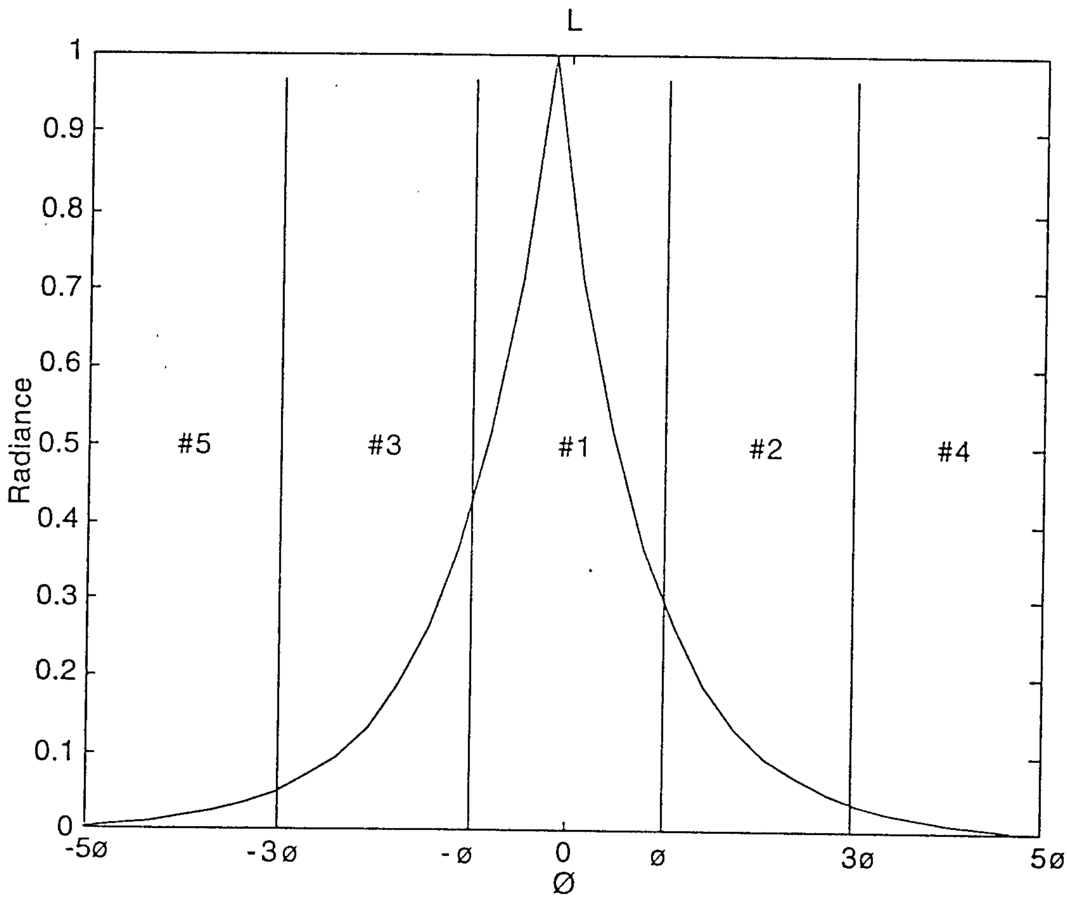

Figure 3. The nonuniform illumination of the instantaneous-fields-of-view (IFOVs) of widths 2ø of a linear array of five numbered detectors by the radiance of eq. (9), vs. angle Ø.

is smaller than the IFOV of the individual detectors then relatively large percent differences can be expected between the measured and interpreted values of radiance.

Similar calculations for when the BW = 2.5 (IFOV), which corresponds to an approximate value k = 0.277, show smaller errors as would be expected for a wider beamwidth. For example the error at the center of the first detector, an area where the radiance is changing most rapidly, is only about 12.7 percent

Table 1. The actual, inferred and percent difference between normalized radiance values when k = 1 which corresponds to a BW = 0.69 (IFOV).

Spatial Location	Actual Normalized Radiance	Inferred Normalized Radiance	Percent Difference
Center of 1st detector	1.0	0.6321	-36.8%
Center of 2nd and 3rd detectors	0.1353	0.1590	+17.5%
Center of 4th and 5th detectors	0.0183	0.0215	+17.5%

while at the centers of the symmetrically located 2nd and 3rd detectors, the error is about 1.3 percent.

IV. ANALYSIS IN THE FIELD-OF-VIEW DOMAIN

From a certain point of view eq. (1) may be regarded as a definition of radiance ($L = dE / d\Sigma$) and it is clearly a point function, yet the measurements are made with apertures, detectors and projected-solid angles for the IFOVs that may be small but are far from being infinitesimal. We will recognize this in our procedures and make use of the mathematical definition of a derivative as a limit

$$L(\Sigma) = dE / d\Sigma = \lim_{\Delta\Sigma \to 0} (E[\Sigma + \Delta\Sigma] - E[\Sigma]) / \Delta\Sigma \tag{10}$$

which can be interpreted as representing a sequence of "non-local" measurements converging to a local result at a specific Ø and a specific Ω. The mathematics literature observes that the ratio in eq. (10) can usually be replaced without concern by the ratio of $\{ E [\Sigma + \Delta\Sigma] - E [\Sigma - \Delta\Sigma] \}$ to $\{ 2 \Delta\Sigma \}$ when that is useful to do.

This procedure is illustrated for the radiance field described in Section III for $k = 1$ for detector number one. Using that detector as the central one in a linear contiguous cluster of five identical detectors (see Figure 3) it can be seen that in the field of view spanned by the five detectors, which is 5 x (2ø) = 10 ø, the total irradiance illuminating the cluster is the sum

$$E5 = E1 + E2 + E3 + E4 + E5 = (2ø)\, L_0\, \{0.6321 + (2)\, 0.1590 + (2)\, 0.02152\}, \tag{11}$$

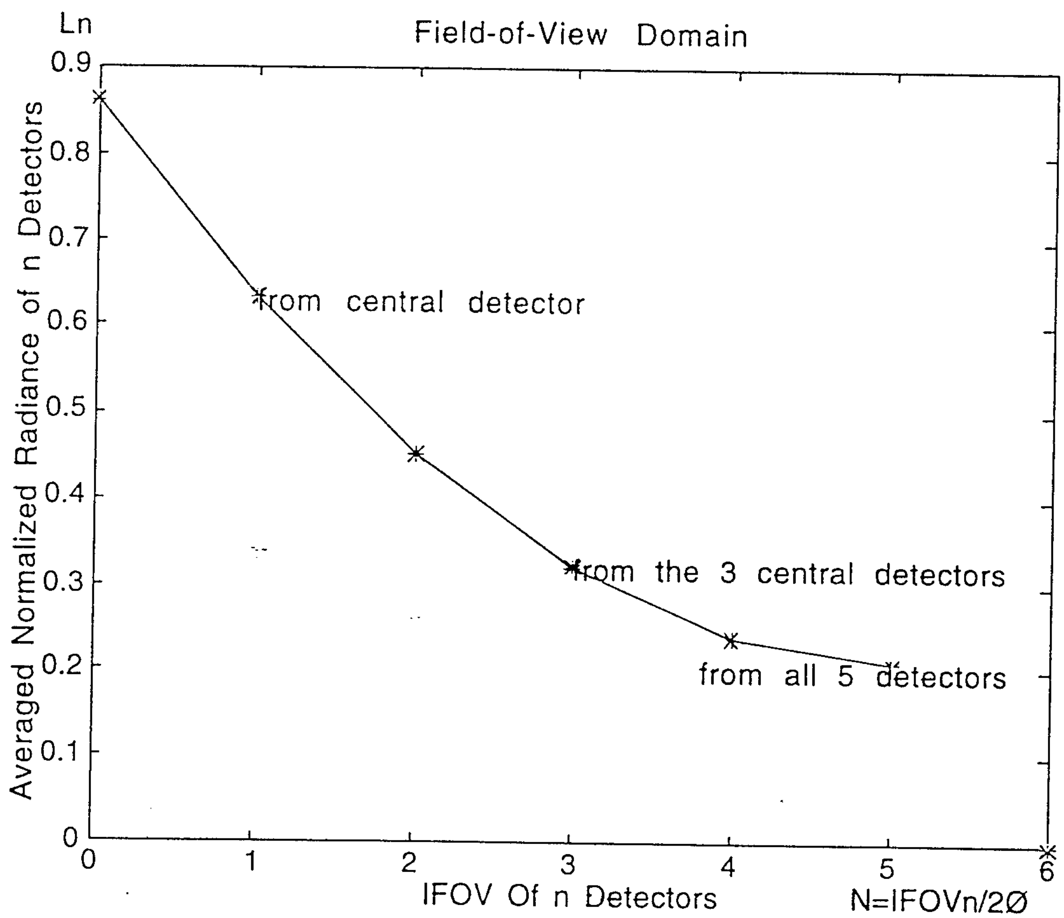

Figure 4. The averaged normalized radiance inferred from the measurements of n detectors vs. the sum of the instantaneous-fields-of-view (IFOVn) of n detectors normalized to the IFOV of one, for Ø = 0 of Figure 3.

where the symmetry of the array accounts for the doubling of 0.1590 and 0.02152, E_i (i = 1, 2, 3, 4, 5) are the irradiances measured at the i th detector and the total yields $E_5 = (2ø) L_0 (0.9941)$. The corresponding average radiance, normalized to L_0, determined by $E_5 / (10 ø)$ is $L_5 = 0.1988$.

In the smaller cluster of three detectors, where number one is the central one, the field of view spanned by the detectors is 6ø and the total illuminating irradiance is $E_3 = (2ø) L_0 (0.6321 + (2) 0.1590)$ corresponding to an average normalized radiance of $E_3 / (6 ø) = L_3 = 0.3170$. The next and smallest cluster in the set of nested clusters is simply the central detector itself for which the average normalized radiance is simply $L_1 = 0.6321$. In this procedure these average normalized radiances are plotted in a field-of-view (FOV) plane with coordinates (FOV_n , L_n) where FOV_n is the total FOV simultaneously observed by the n contiguous, linearly arrayed detectors and L_n is the normalized radiance as calculated for the total averaged radiance seen in the FOV of the n detectors as shown in Figure 4.

The final step in the procedure is to extrapolate through those plotted points to the limit where the IFOV = 0 (see Figure 4). In most work, limits are obtained through the examination and manipulation of algebraic expressions, however now the analytic relations are unknown and some sort of graphical or equivalent procedure must be employed. Here a simple curve-fit has been used where a quadratic polynomial has been chosen which passes through the plotted points. The result obtained was

(12) $$L_n(N) = 0.864 - 0.256 (N) +0.025 (N)^2,\quad N = FOV_n / (2ø)$$

Clearly the result of interest is that for N = 0 (corresponding to IFOV = 0). This can be seen from eq. (12) to be $L_n = 0.864$ which obviously represents a more accurate approximation to the true normalized radiance value of 1.000 at Ø = 0 than the measured (really inferred from the measurement) normalized radiance 0.632 (from an average over the full IFOV of the central detector) to the peak value of radiance seen in the central detector along the line-of-sight normal to the geometric (linear) center of the detector surface. Since we have been dealing with this example as one-dimensional, that geometric center is a line not a point.

The preceding procedure is appropriate for peaks, valleys and stretches of monotonically increasing or decreasing radiance data. When a segment is "almost" monotonic, i.e., a peak or valley lies near the end of the segment, some adjustment, such as the split-field decomposition is required. In the example that has been explored this may be best illustrated by considering the (inferred) data point L = 0.1590 at Ø = -2ø which is at the center of detector number three. To calculate a (corrected) limit radiance value at this point, the data from detectors numbered one, three, and five will be used (see Figure 3). However the full irradiance from detector number one cannot be used because the downturn in radiance illumination just past the peak at Ø = 0 is not "relevant" to the monotonically increasing curve just before the peak. Only the half on the side of the curve of interest can be used. Thus the irradiances within the FOV from Ø = -5ø to Ø = 0 will be used.

First observe that the total sum from the three detectors will read

(13) $$E_3 = E4 + E3 + (1/2) E1 = (2ø) L_o \{0.0215 + 0.1590 + 0.6321/2\}.$$

From this the averaged, normalized radiance in the FOV = 5ø is then calculated to be

(14) $$Lav_3 = 0.1986$$

Eq. (14) along with $Lav_1 = 0.1590$ can be plotted in the FOV plane at N = 2.5 and 1.0 respectively and a

Table 2. The actual, measured, corrected and percent difference between actual and corrected normalized radiance values when k = 1.

Spatial Location	Actual Radiance	Inferred Radiance	Corrected Radiance	Percent Difference
Center of 1st detector	1.0	0.6321	0.8638	13.6%
Center of 2nd and 3rd detectors	0.1353	0.1590	0.1326	2.0%

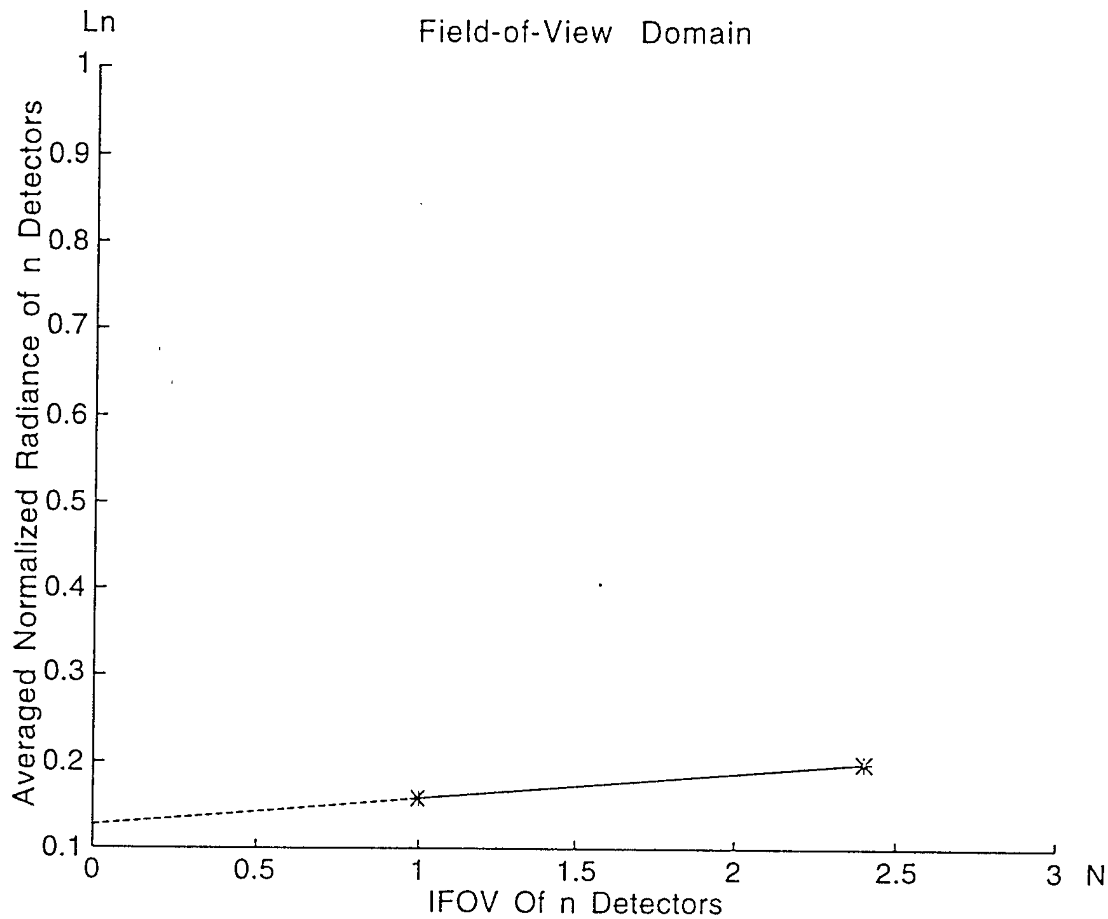

Figure 5. Average, normalized radiance summed from n detectors vs. the sum of the IFOVs of n detectors (IFOVn) divided by the IFOV of one detector, for Ø = - 2ø of Figure 3 using the split-view approach.

linear curve-fit through them can be constructed (see Figure 5). It is

(15) $L_n(N) = 0.1326 + 0.0264\ (N)$,
$N = FOV_n / 2ø$

Once again it is clear, based on the overall strategy, that N = 0 is required and the result is $L_n(0) = 0.13264$, a result only in error by 2 percent (down from 17.5 percent) compared with the true value of $L(-2ø) = 0.135335$. From all of the proceeding calculations of corrections, a brief table, Table 2, has been constructed.

It can be seen by comparing Tables 1 and 2 that the correction of the peak value by the method proposed in this paper has reduced the error from 36.8 percent to 13.6 percent while the error at the centers of the second and third detectors have been reduced from 17.5 percent to 2.0 percent.

V. ENHANCEMENT OF SPATIAL RESOLUTION

The increased accuracy shown in the preceding results has some relation to an improvement in spatial resolution. There are a number of measures of spatial resolution in current use (see Tables 12-2 and 12-3 of Holst, 1995) but the McKenna equivalent resolution enhancement factor M_r is probably most useful here (McKenna, 1997). It is related to the factor F by which, for the same radiance beamwidth, the IFOV would have to be smaller in order for the usual method of inference of radiance from the measurement process to produce the same accuracy as the corrective algorithm produces from the original measurement data. It can be defined by $M_r = 1 / \{F\}$ where F is expressed as a decimal.

For the radiance described by eq. (9), for when k = 1, we must look for the decrease in IFOV width for which the original measured radiance value

of 0.632 (normalized) would now be measured as 0.864 with same radiance illumination beamwidth. In evaluating the average value of L(Ø) a new angular width of integration of 2aø must be used (a < 1) and the question is: For what value of "a" does the average yield 0.864 ? The answer turns out to be a = 0.300. Thus a 70 percent reduction in IFOV (and equivalent pixel) width, equivalent to F = 0.3, is required for each detector of the array in order for the accuracy of the measurement of the peak value of the radiance field to duplicate the effect of the corrective algorithm. Then the resolution enhancement factor is $M_r = 3.33$.

It also can be shown that the error in the measured value of the radiance at the center of the second and symmetrically located third detector (for k = 1 and a = 0.300) is reduced to only 0.83 percent from 17.5 percent. This reduction in error (a factor of about 1/21) is far greater than that for the peak (a factor of about one-third) but because the radiance is dropping/rising more slowly here than at the peak, a larger reduction in error should be expected.

VI. CONCLUSIONS

This paper has addressed a source of image distortion or error that does not appear to be directly considered in the literature and has presented a technique that appears able to reduce that error substantially when the error or distortion is not small. The source of the distortion is the unknown nonuniform illumination of the IFOV of individual detectors in detector arrays, each of which outputs some averaged value for each wavelength band covered by those detectors. These distortions currently do not appear to be calibrated out of the imagery by pre- or post-operational calibrations. Nor is it clear that such calibration is even possible in principle.

The technique used to correct that distortion, if only partially, is based on replacing the derivative relation $L = dE / d\Sigma$ with the mathematically equivalent limit that is then evaluated operationally in the FOV domain using nested clusters of detectors that surround the detector of interest. The limit is obtained from a sequence of values of irradiance, E_n, that is measured by the totality of detectors within a cluster and from which an effective radiance, L_n, is derived based on the total projected solid-angle viewed by the cluster. As the sequence progresses to smaller clusters and hence smaller viewed FOV, the true radiance seen through an IFOV of zero width is approached.

This is the desired result, however the quantization by the very nature of real world detectors in the nested clusters into a finite number of non-infinitesimal sized devices introduces errors that prevent the experimentally determined sequence in the FOV domain from exactly yielding the precise limit, except possibly in special circumstances. We have used both a split-field (when the last detector in a sequence for a monotonically increasing/decreasing set of irradiance measurements is viewing the peak of the curve within the IFOV) and full-field approaches.

This paper has limited its focus to a simple analytical model of the illuminating radiance for its calculations. Miller and Friedman have noted that optical trackers have demonstrated the ability to locate targets (i.e. signal peaks) to an accuracy of 1 /100 of a pixel and better (Miller and Friedman 1996) using the method of centroids. Thus, location of the radiance peak, if there is only one, within the pixel is not a barrier at this point in time to further progress with the derivative-as-limit approach.

This represents the beginning of our work in this area. Initial calculations with less ideal models (e.g. no symmetries) using one-sided limits as well as with algorithms unrelated to the derivative-as-limit approach used here, show comparable, possibly even greater, promise. Clearly the presence of other sources of errors (and there are many) must affect these results, however it seems there are many circumstances where such effects will not be dominant. Comparison of the initial and algorithmically improved images shown in Figures 1 and 2, respectively, seem to bear this out.

The imagery and calculations indicate that using the new approach may result in improved contrast, spatial deblurring and spatial resolution (in the sense of the McKenna criterion). This should apply to all of the spectral bands and results in color composite images that appear superior to the human eye compared to single spectral band images.

VII. REFERENCES

Dereniak, E. L. and G. D. Boreman, "Infrared Detectors and Systems," John Wiley, NY, 1996.

Holst, G. C., "Electro-Optical Imaging System Performance," SPIE Opt. Eng. Press, 1995.

Kaufman, Y. J., "The atmospheric effect on remote sensing and its correction," Chap. 9 of "Theory and Applications of Optical Remote Sensing," G. Asrar, Ed., John Wiley, NY, 1989.

McKenna, Charles M., Personal communication, May 1997.

Miller, J. L. and E. Friedman, "Photonics Rules of Thumb: Optics, Electro-Optics, Fiber Optics, and Lasers, " McGraw-Hill, NY, p. 50, 1996.

Nicodemus, F. E., J. C. Richmond, et al., "Geometrical considerations and nomenclature in reflectance," National Bureau of Standards, U.S. Dept. Commerce, 1977.

Nussbaum, A. and R. A. Phillips, "Contemporary Optics for Scientists and Engineers," Prentice-Hall, Englewood Cliffs, NJ, Chap. 10, 1976.

Schulz, M. and L. Caldwell, "Nonuniformity correction and correctability of infrared focal plane arrays," Infrared Phys. & Tech., 36, 1995, 763-777.

Stover, J. C., "Optical Scattering: Measurement and Analysis," 2nd ed., SPIE Opt. Eng. Press, 1995.